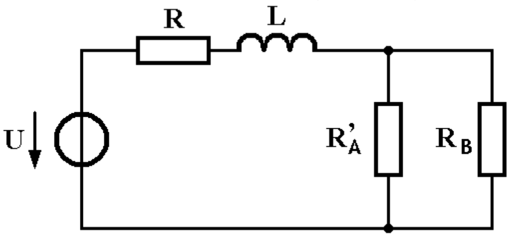
SOLUTION SET IX

EXERCISE IX.1 : BALANCED THREE-PHASE SYSTEM

We replace user A with an equivalent wye user. R_A resistors become: $R'_A = R_A/3 = 10{,}8/3 = 3{,}6\ \Omega$.

The system being symmetrical, no current flows in the neutral. The resistor R_N causes no voltage drop and calculations can be made by directly connecting the neutrals of the users to the neutral of the generator. In addition, the calculation can be limited to a single phase according to the diagram opposite.



By putting in parallel R'_A and R_B, we obtain the equivalent resistance :

$$R_{éq} = \frac{R'_A \cdot R_B}{R'_A + R_B} = 1{,}8\ \Omega$$

The line resistance and inductance are:

$$R = R' \cdot d = 0{,}2\ \Omega \qquad L = L' \cdot d = 4\ \text{mH}$$

The generator delivers to an impedance per phase:

$$\underline{Z} = R + R_{éq} + j\omega L = (2 + j\,1{,}257)\ \Omega$$

$$|\,\underline{Z}\,| = Z = \sqrt{2^2 + 1{,}256^2} = 2{,}362\ \Omega \qquad \arg(\underline{Z}) = \text{arctg}\frac{1{,}256}{2} = 32{,}14°$$

A. The current discharged by one phase of the generator will be :

$I = \dfrac{U}{Z} = \underline{100\ A}$, behind of 32,14° on the phase voltage at its terminals.

B. The active power provided by the generator is worth :

for a phase : $P_{1load} = U \cdot I \cos\phi = \underline{20\ kW}$

for the three phases : $P_{3loads} = 3 \cdot P_{1load} = \underline{60\ kW}$

The reactive power provided by the generator is worth :

for a phase : $Q_{1load} = U \cdot I \sin\phi = \underline{12{,}5\ k\,\text{var}}$

for the three phases : $Q_{3loads} = 3 \cdot Q_{1load} = 3 \cdot 12{,}53 = \underline{37{,}7\ k\,\text{var}}$

C. Active power consumed

☞ **Both users consume the same active power because they have the same wye resistance.**

for a phase : $P_{1load_A} = P_{1load_B} = \frac{1}{2}(R_{eq} \cdot I^2) = \underline{9\ kW}$

for the three phases : $P_{3load_A} = P_{3load_B} = 3 \cdot P_{load_A} = 27\ kW$

☞ **Users A and B, being purely resistive, they consume no reactive power.**

EXERCISE IX.2 : COMPENSATION OF THE REACTIVE IN THREE-PHASE SYSTEMS

A. Total reactive power is given by :

$$Q = 3 \cdot U_Z I_Z \sin\phi - 3 \cdot U_C I_C$$

with : U_Z, I_Z = phase voltage and phase current on each inductive load Z

U_C, I_C = phase voltage and phase current on each capacitor C

Compensation is obtained when :

$$Q = 0 \quad \Leftrightarrow \quad U_Z I_Z \cdot \sin\phi = U_C I_C$$

The inductive load is connected in triangle. So we have relationships :

$U_Z = U_l$ et $I_Z = \dfrac{\sqrt{3} \cdot U}{Z} = \dfrac{U_{lig}}{Z}$ (where U is the rms value of the phase voltage)

The capacitors are connected in star. So we have relationships :

$U_C = U = U_l / \sqrt{3}$ et $I_C = \dfrac{U}{Z_C} = \dfrac{U_l}{\sqrt{3}} \cdot C\omega$ (where Z_C = impedance of a capacitor)

From there, the condition of cancellation of reactive power is written :

$$U_l \cdot \frac{U_l}{Z} \cdot \sin\varphi = \frac{U_l}{\sqrt{3}} \cdot \frac{U_l}{\sqrt{3}} \cdot C\omega \quad \Rightarrow \quad C = \frac{3\sin\phi}{\omega Z} = \underline{573\ \mu F}$$

B. $P = 3 \cdot U_Z I_Z \cos\varphi = 3 \cdot U_l \dfrac{U_l}{Z} \cdot \cos\varphi = \underline{38{,}4\ kW}$

C. In all cases, the active power is given as a function of the line voltage and line current, by :

$$P = \sqrt{3} \cdot U_l I_l \cos\varphi$$

<u>Without</u> the capacitors, $\cos\phi = 0{,}8 \quad \Rightarrow \quad I_l = \underline{69{,}28\ A}$

<u>With</u> the capacitors, the active power is the same (the capacitors do not consume active power) and $\cos\phi = 1 \quad \Rightarrow \quad I_l = \underline{55{,}43\ A}$